Q. No. 2 Part (i) **Cube roots of unity:**

$$\omega^{40} + \omega^{50} + 1 = 0$$

Left Hand side:

$= \omega^{40} + \omega^{50} + 1$

$= \omega^{39} \cdot \omega^{1} + \omega^{48} \cdot \omega^{2} + 1$

$= (\omega^{3})^{13} \cdot \omega^{1} + (\omega^{3})^{16} \cdot \omega^{2} + 1$

$= (1)^{13} \cdot \omega + (1)^{16} \cdot \omega^{2} + 1$ $\quad [\because \omega^{3} = 1]$

$= 1 \cdot \omega + 1 \cdot \omega^{2} + 1$

$= \omega + \omega^{2} + 1$

$= 1 + \omega + \omega^{2}$ $\quad [\because 1 + \omega + \omega^{2} = 0]$

$= 0$

Hence Proved

$L.H.S = R.H.S$

Q. No. 2 Part (ii) **Sets :**

$U = \{1, 2, 3, ---, 10\}$

$P = \{2, 3, 4, 5, 8\}$ $\quad$ $Q = \{1, 3, 5, 7, 9\}$

To verify :

$$(P - Q)' = P' \cup Q$$

L.H.S :

$P - Q = \{2, 3, 4, 5, 8\} - \{1, 3, 5, 7, 9\}$

$= \{2, 4, 8\}$

$(P - Q)' = U - (P - Q)$

$= \{1, 2, 3, --, 10\} - \{2, 4, 8\}$

$= \{1, 3, 5, 6, 7, 9, 10\}$

R.H.S :

$P' = U - P$

$= \{1, 2, 3, 4, ---10\} - \{2, 3, 4, 5, 8\}$

$= \{1, 6, 7, 9, 10\}$

$P' \cup Q = \{1, 6, 7, 9, 10\} \cup \{1, 3, 5, 7, 9\}$

$= \{1, 3, 5, 6, 7, 9, 10\}$

Hence Proved

L.H.S = R.H.S

Q. No. 2 Part (iii) **Trigonometric Identities:**

To verify :

$$\frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta$$

L.H.S :

$$= \frac{1}{\sec\theta - \tan\theta}$$

$$= \frac{1}{\sec\theta - \tan\theta} \times \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$$

$$= \frac{\sec\theta + \tan\theta}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}$$

$$= \frac{\sec\theta + \tan\theta}{(\sec\theta)^2 - (\tan\theta)^2} \qquad [\because (a-b)(a+b) = a^2 - b^2]$$

$$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$$

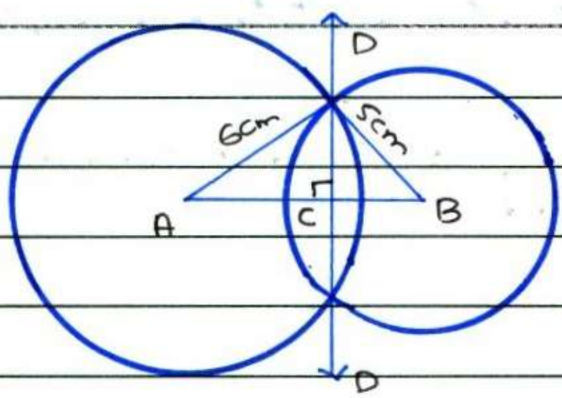
$$= \frac{\sec\theta + \tan\theta}{1} \qquad [\because 1 + \tan^2\theta = \sec^2\theta$$
$$1 = \sec^2\theta - \tan^2\theta]$$

$$= \sec\theta + \tan\theta$$

Hence Proved

L.H.S = R.H.S

Q. No. 2 Part (iv)



Given: $m\overline{AD} = 6cm$, $m\overline{BD} = 5cm$

$m\overline{CD} = 4cm$

To find: $m\overline{AB} = ?$

Solution: As ΔACD is a right triangle so,

$(AD)^2 = (AC)^2 + (CD)^2$ (Pythagoras theorem)

$(6)^2 = (AC)^2 + (4)^2$

$m\overline{AC} = \sqrt{6^2 - 4^2}$

$m\overline{AC} = 2\sqrt{5}$ cm OR

4.47cm

Similarly, In Δ BCD

$(BD)^2 = (CB)^2 + (CD)^2$

$(5)^2 = (CB)^2 + (4)^2$

$m\overline{CB} = \sqrt{5^2 - 4^2}$

$m\overline{CB} = 3$ cm

$m\overline{AB} = m\overline{AC} + m\overline{CB}$

$m\overline{AB} = (2\sqrt{5} + 3)$ cm

$m\overline{AB} = (3 + 2\sqrt{5})$ cm OR

$m\overline{AB} = 7.47$ cm

The distance between the centres A & B is 7.47cm

Q. No. 2 Part (v) **Roots of Equation:**

$$x^2 - 3x + 6 = 0$$

α, β are roots of above equation.

$\Rightarrow$ Sum of roots $= \alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3$

$\Rightarrow$ Product of roots $= \alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$

Roots of new equation : $\alpha^2\beta$ and $\alpha\beta^2$

$\Rightarrow$ Sum of roots of new eq $= \alpha^2\beta + \alpha\beta^2$

$$= \alpha\beta(\alpha + \beta)$$
$$= 6(3)$$
$$= 18$$

Now, the sum of roots of the new equation that we will form is 18. Similarly, we will find the product of the roots:

$\Rightarrow$ Product of roots of new eq $= (\alpha^2\beta)(\alpha\beta^2)$

$$= \alpha^3\beta^3$$
$$= (\alpha\beta)^3$$
$$= (6)^3$$
$$= 216$$

Equation: $x^2 - Sx + P = 0$

$$x^2 - (+18)x + 216 = 0$$
$$x^2 - 18x + 216 = 0$$

$x^2 - 18x + 216$ is the equation whose roots are $\alpha^2\beta$ and $\alpha\beta^2$

Q. No. 2 Part (vi) **Exponential Equation:**

$$4(2^{2x+1}) - 9(2^{x}) + 1 = 0$$

$$4\times(2^{2x} \cdot 2^{1}) - 9(2^{x}) + 1 = 0$$

$$4\times 2(2^{2x}) - 9(2^{x}) + 1 = 0$$

$$8(2^{2x}) - 9(2^{x}) + 1 = 0$$

Let $2^{x} = y$ and $2^{2x} = y^2$

$$8y^2 - 9y + 1 = 0$$

$$8y^2 - 8y - y + 1 = 0$$

$$8y(y-1) - 1(y-1) = 0$$

$$(y-1)(8y-1) = 0$$

$\Rightarrow y - 1 = 0$ $\qquad\qquad$ $\Rightarrow 8y - 1 = 0$

$y = 1$ $\qquad\qquad$ $y = \frac{1}{8}$

Put $y = 2^{x}$

$\Rightarrow y = 2^{x} = 1$ $\qquad\qquad$ $\Rightarrow y = 2^{x} = \frac{1}{8}$

$2^{x} = 2^{0}$ $\qquad\qquad$ $2^{x} = \frac{1}{2^3}$

$\Rightarrow x = 0$

$2^{x} = 2^{-3}$

$\Rightarrow x = -3$

Solution Set = $\{-3, 0\}$

Q. No. 2 Part (vii) **Componendo Dividendo :**

$$\frac{a^2+b^2}{a^2-b^2} = \frac{ac+bd}{ac-bd}$$

Solution

$$\frac{(a^2+b^2)+(a^2-b^2)}{(a^2+b^2)-(a^2-b^2)} = \frac{(ac+bd)+(ac-bd)}{(ac+bd)-(ac-bd)}$$

$$\frac{a^2+\cancel{b^2}+a^2-\cancel{b^2}}{\cancel{a^2}+b^2-\cancel{a^2}+b^2} = \frac{ac+\cancel{bd}+ac-\cancel{bd}}{\cancel{ac}+bd-\cancel{ac}+bd}$$

$$\frac{\cancel{2}a^2}{\cancel{2}b^2} = \frac{\cancel{2}ac}{\cancel{2}bd}$$

$$\frac{a^2}{b^2} = \frac{ac}{bd}$$

By Alternendo theorem :

$$\frac{a^{\cancel{2}}}{\cancel{a}c} = \frac{b^{\cancel{2}}}{\cancel{b}d}$$

$$\frac{a}{c} = \frac{b}{d}$$

Again by Alternendo theorem :

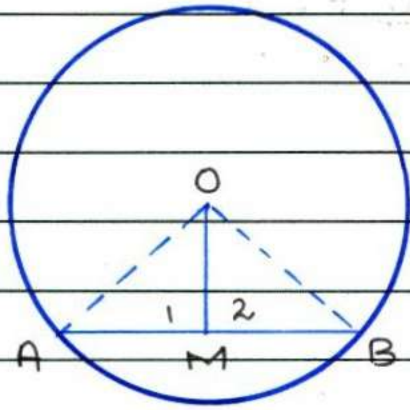
$$\frac{a}{b} = \frac{c}{d}$$

OR

$$a:b = c:d$$

Q. No. 2 Part (viii) **Theorem:**

figure:



Given: $\overline{AB}$ is the chord of a circle with centre O and M is the midpoint of a chord. $m\overline{AM} = m\overline{BM}$

To Prove: $\overline{OM}$ is perpendicular to chord,

$\overline{OM} \perp \overline{AB}$

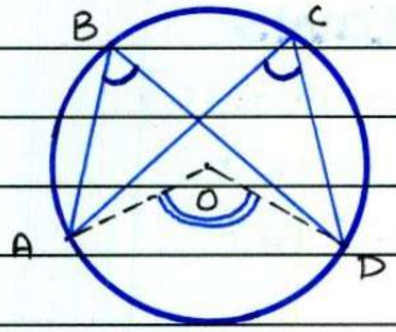
Construction: Join O to A and B so we form 2 triangles. Label the angles a $\angle 1$ and $\angle 2$

Proof:

Statements	**Reasons**
In $\triangle AOM \leftrightarrow \triangle BOM$	
$m\overline{OM} = m\overline{OM}$	Common
$m\overline{OA} = m\overline{OB}$	Radii of same circle
$m\overline{AM} = m\overline{BM}$	Given.
So, $\triangle OAM \cong \triangle OBM$	S·S·S Postulate (S·S·S ≅ S·S·S)
Hence,	
$m\angle 1 = m\angle 2$ (i)	Corresponding angles of congruent $\triangle$s.
But	
$m\angle 1 + m\angle 2 = 180^\circ$ (ii)	Adjacent supplementary angles.
$m\angle 1 = m\angle 2 = 90^\circ$	Using (i) and (ii)
Hence,	
$\overline{OM}$ is perpendicular to chord $\overline{AB}$	

Q. No. 2 Part (ix)

figure :



Given : ∠ABD and ∠ACD are 2 angles in the same segment of a circle with centre O.

To Prove : m∠ABD = m∠ACD

Construction : Join A to O and D to O so that ∠AOD is the central angle.

Proof :

Statements	Reasons
A circle with centre has ∠ABD and ∠ACD circum angles in same segment and ∠AOD is central angle	Given and construction.
m∠AOD = 2m∠ABD (i)	The measure of central angle of a minor arc is double that of angle subtended by corresponding
m∠AOD = 2m∠ACD (ii)	major arc. (Same reason for (i) & (ii))
$\cancel{2}$m∠ABD = $\cancel{2}$m∠ACD	from (i) and (ii)
m∠ABD = m∠ACD	

Class	f	x	f/x
31-40	28	35.5	0.788
41-50	18	45.5	0.395
51-60	10	55.5	0.180
61-70	14	65.5	0.213
71-80	30	75.5	0.397

$\Sigma f = 100$ $\Sigma f/x = 1.973$

Harmonic mean $= \dfrac{\Sigma f}{\Sigma \frac{f}{x}}$

$= \dfrac{100}{1.973}$

$= 50.68$

Q. No. 3 (Page 1)

De Morgan's Laws

$U = \{x \mid x \in N \wedge 1 \le x \le 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$P = \{x \mid x \in E \wedge 1 < x \le 10\}$

$= \{2, 4, 6, 8, 10\}$

$Q = \{x \mid x \in P \wedge 1 < x < 10\}$

$= \{2, 3, 5, 7\}$

(i) $(P \cup Q)' = P' \cap Q'$

L·H·S :

$P \cup Q = \{2, 4, 6, 8, 10\} \cup \{2, 3, 5, 7\}$

$= \{2, 3, 4, 5, 6, 7, 8, 10\}$

$(P \cup Q)' = U - (P \cup Q)$

$= \{1, 2, 3, --- 10\} - \{2, 3, 4, 5, 6, 7, 8, 10\}$

$= \{1, 9\}$

R·H·S :

$P' = U - P = \{1, 2, 3 --- 10\} - \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

$Q' = U - Q = \{1, 2, 3 --- 10\} - \{2, 3, 5, 7\}$

$= \{1, 4, 6, 8, 9, 10\}$

$P' \cap Q' = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9, 10\}$

$= \{1, 9\}$

Hence Proved

L·H·S = R·H·S

(ii) $(P \cap Q)' = P' \cup Q'$

L·H·S :

$P \cap Q = \{2, 4, 6, 8, 10\} \cap \{2, 3, 5, 7\}$

$= \{2\}$

$(P \cap Q)' = U - (P \cap Q)$

$= \{1, 2, 3 --- 10\} - \{2\}$

$= \{1, 3, 4, 5, 6, 7, 8, 9, 10\}$

R.H.S :

$P' = U - P = \{1, 2, 3 --- 10\} - \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

$Q' = U - Q = \{1, 2, 3 --- 10\} - \{2, 3, 5, 7\}$

$= \{1, 4, 6, 8, 9, 10\}$

$P' \cup Q' = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9, 10\}$

$= \{1, 3, 4, 5, 6, 7, 8, 9, 10\}$

Hence Proved

L.H.S = R.H.S

Q. No. 4 (Page 1) **Simultaneous Equations:**

$x^2 + y^2 = 20$ -- (i)

$6x^2 + xy - y^2 = 0$ -- (ii)

From eq (ii)

$6x^2 + xy - y^2 = 0$

$6x^2 + 3xy - 2xy - y^2 = 0$

$3x(2x + y) - y(2x + y) = 0$

$(2x + y)(3x - y) = 0$

$\Rightarrow 2x + y = 0$ $\Rightarrow 3x - y = 0$

$y = -2x$ --(1) $y = 3x$ -- (2)

Put value of y from (1) in (i)

$x^2 + (-2x)^2 = 20$

$x^2 + 4x^2 = 20$

$5x^2 = 20$

$x^2 = \frac{20}{5}$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

Now, put $x = \pm 2$ in (1)

For $x = 2$: $y = -2(2)$ For $x = -2$: $y = -2(-2)$

$y = -4$ $y = 4$

Similarly put value of y from (2) in (i)

$x^2 + (3x)^2 = 20$

$x^2 + 9x^2 = 20$

$10x^2 = 20$

$\sqrt{x^2} = \sqrt{2}$

Q. No. 4 (Page 2)

$x = \pm\sqrt{2}$

put value of x in (2)

$y = 3x$

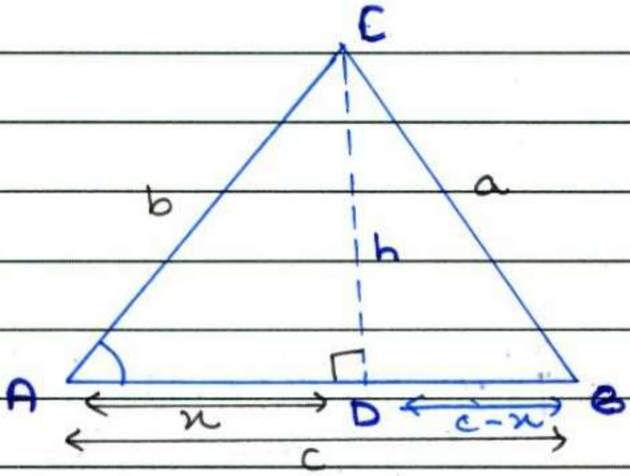
$y = 3(\pm\sqrt{2})$

$y = \pm 3\sqrt{2}$

Solution Set = $\{(2,-4), (-2,4), (+\sqrt{2}, 3\sqrt{2}), (-\sqrt{2}, -3\sqrt{2})\}$

Q. No. 5 (Page 1) Theorem:

Figure:



Given:

$\triangle ABC$ has an acute angle $\angle CAB$ acute at A.

Take $m\overline{AC} = b$ $\quad m\overline{BC} = a$

$m\overline{AB} = c$

Draw $\overline{CD}$ perpendicular to $\overline{AB}$ so that $\overline{AD}$ is the projection of $\overline{AC}$ upon $\overline{AB}$

Also,

Take $m\overline{AD} = x$

$m\overline{CD} = h$

To Prove:

$$(BC)^2 = (AC)^2 + (AB)^2 - 2(m\overline{AB})(m\overline{AD})$$

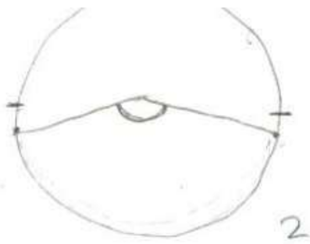
$$a^2 = b^2 + c^2 - 2cx$$

Proof:

Statements	Reasons
In $\angle$rt $\triangle ADC$	
$m\angle CDA = 90^\circ$	Given.
So,	
$(AC)^2 = (AD)^2 + (CD)^2$	Pythagoras theorem.
$b^2 = x^2 + h^2 \quad$ (i)	

Q. No. 5 (Page 2)

Similarly,	
In $\angle$rt Δ CDB,	
$m\angle CDB = 90^\circ$	Given.
$(CB)^2 = (CD)^2 + (BD)^2$	Pythagoras theorem.
$a^2 = h^2 + (c-x)^2$	
$a^2 = h^2 + c^2 + x^2 - 2cx$	
$a^2 = h^2 + x^2 + c^2 - 2cx$ (ii)	
$a^2 = b^2 + c^2 - 2cx$	from (i) and (ii)
Hence,	
$(BC)^2 = (AC)^2 + (AB)^2 - 2(m\overline{AB})(m\overline{AD})$	



$4x^2 - 3x + 7$

$\alpha + \beta = \frac{-b}{a}$

$\alpha\beta = \frac{c}{a}$ $\frac{7}{4}$ $\frac{1}{ab} = \frac{a}{c}$ $\frac{4}{7}$

$2a+1 : 21 :: 4 \cdot 7$

$8^2 - 4(16)(11)$ $7(2a+1) = 21 \times 4$

$14a + 7 = 84$

$-9 \pm 3\sqrt{5}i$

$x = 1$

77

$\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)} = \frac{-A}{2(x+1)} + \frac{B}{2(x-1)}$ $x = -1$

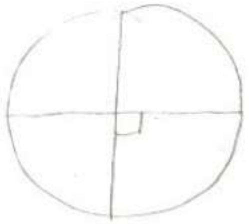
$1 = A(x-1) + B(x+1)$

$\frac{3 \pm \sqrt{15}i}{2}$

$\boxed{\frac{1}{2} = B}$ $A = \frac{-1}{2}$

$\frac{1}{2(x-1)} - \frac{1}{2(x+1)}$

$\frac{\cancel{x}+1 - \cancel{x}+1}{2(x+1)(x-1)}$



$2x + 3 = 4x$

$3 = 4x - 2x$

$3 = 2x$

$\frac{3}{2} = x$

$X = t$

$x \times X$

$\frac{3 + x + 1}{2} t^2$

$\bar{x} = 32.5$

~~$\Sigma fx - \Sigma f$~~

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$32.5 = \frac{1300}{\Sigma f}$

t

$n(x) = 3$

9

$x = \{1, 2, 3\}$